A jövő internete, BMEVITMAV74

BME-VIK és DE-IK közös szabadon választható tárgya

Új módszerek a Jövő Internet modellezésében

Prof. Sztrik János DSc

Debreceni Egyetem

Informatikai Kar

Informatikai rendszerek és Hálózatok Tanszék

Debrecen, 2015. tavasz

NEW SZÉCHENYI PLAN

FIRST – Future Internet Research, Services and Technology

TÁMOP-4.2.2.C-11/1/KONV-2012-0001

Queueing Theory with Applications
A Personal View

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HUNGARY'S RENEWAL

The project is supported by the European Union and co-financed by the European Social Fund.

Outline

- Origin of Queueing Theory
- Classifications of Queueing Systems
- Applications
- Solution Methods
- Basic Formulas and Laws
- Hungarian Contributions
- Recent Developments
- References

Origin of Queueing Theory

Agner Krarup Erlang, 1878-1929

- "The Theory of Probabilities and Telephone Conversations", Nyt Tidsskrift for Matematik B, vol 20, 1909.
- "Solution of some Problems in the Theory of Probabilities of Significance in Automatic Telephone Exchanges", Elektrotkeknikeren, vol 13, 1917.
- "The life and works of A.K. Erlang", E. Brockmeyer, H.L. Halstrom and Arns Jensen, Copenhagen: The Copenhagen Telephone Company, 1948.

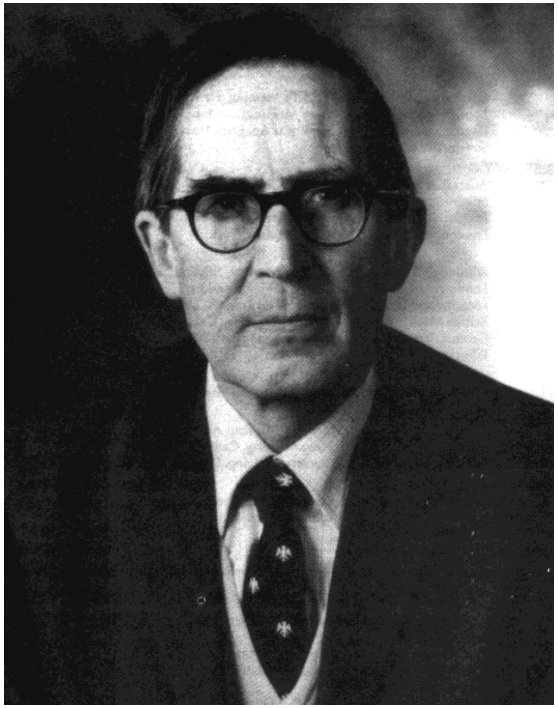
Queueing Theory Homepage

http://web2.uwindsor.ca/math/hlynka/queue.html

Applications

- Telephony
- Manufacturing
- Inventories
- Dams
- Supermarkets
- Computer and Communication Systems
- Call Centers
- Infocommunication Networks
- Hospitals
- Many others

Kendall’s Notation



David G. Kendall, 1918-2007

A/B/c/K/m/Z

Performance Metrics

- Utilizations
- Mean Number of Customers in the System / Queue
- Mean Response / Waiting Time
- Mean Busy Period Length of the Server
- Distribution of Response / Waiting Time
- Distribution of the Busy Period

Solution Methodologies

- Analytical
- Numerical
- Asymptotic
- Simulation
- Tools

Erlang Loss Formula, M/G/c/c Systems

$$B(c,\rho) = p_c = \frac{\rho^c / c!}{\sum_{n=0}^{c} \rho^n / n!} \qquad \rho = \lambda E(B)$$

$$B(c,\rho) = \frac{\rho B(c-1,\rho)/c}{1+\rho B(c-1,\rho)/c} = \frac{\rho B(c-1,\rho)}{c+\rho B(c-1,\rho)}$$

$$B(0,\rho) = 1$$

Approximation Formula

$$B(n,\rho) \approx \frac{\Phi(s) - \Phi(s-1)}{\Phi(s)} = 1 - \frac{\Phi(s-1)}{\Phi(s)},$$

$$\Phi(s) = \int_{-\infty}^{s} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx, \qquad s = \frac{n + \frac{1}{2} - \varrho}{\sqrt{\varrho}}.$$

http://www.erlang.com/calculator/

http://jani.uw.hu/erlang/erlang.html

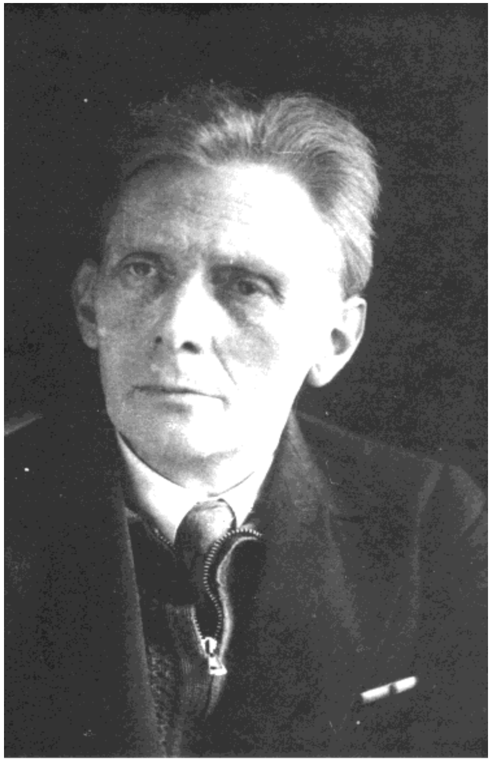
Erlang Delay Formula
M/M/n Systems

$$C(n,\rho) = \frac{nB(n,\rho)}{n-\rho+\rho B(n,\rho)},$$

$$C(n,\rho) = \frac{\rho(n-1-\rho)\cdot C(n-1,\rho)}{(n-1)(n-\rho)-\rho C(n-1,\rho)}, \quad C(1,\rho)=\rho$$

Pollaczek-Khintchine Formulas, M/G/1 Systems

Felix Pollachek, 1892-1981



Alexander Y. Khintchine, 1894-1959

Mean Value Formulas

$$C_X^2 := \frac{\mathrm{Var}[X]}{(E[X])^2}$$

$$E[W] = E[R]\frac{\rho}{1-\rho} = \frac{E[B]}{2}\frac{\rho}{1-\rho}(1 + C_B^2)$$

$$E[T] = E[B]\left(1 + \frac{\rho(1 + C_B^2)}{2(1-\rho)}\right)$$

Transform Formulas

$$G_N(z) = L_B(\lambda(1-z)) \cdot \frac{(1-\rho)(1-z)}{L_B(\lambda(1-z)) - z}$$

$$L_T(s) = L_B(s) \frac{s(1-\rho)}{s - \lambda + \lambda L_B(s)}$$

Little's Law

$$\mathbb{E}(N) = \bar{\lambda}\mathbb{E}(T)$$

$$\mathbb{E}(Q) = \bar{\lambda}\mathbb{E}(W)$$

$$\mathbb{E}(N(N-1)\dots(N(-k+1))) = \lambda^k\mathbb{E}(T^k)$$

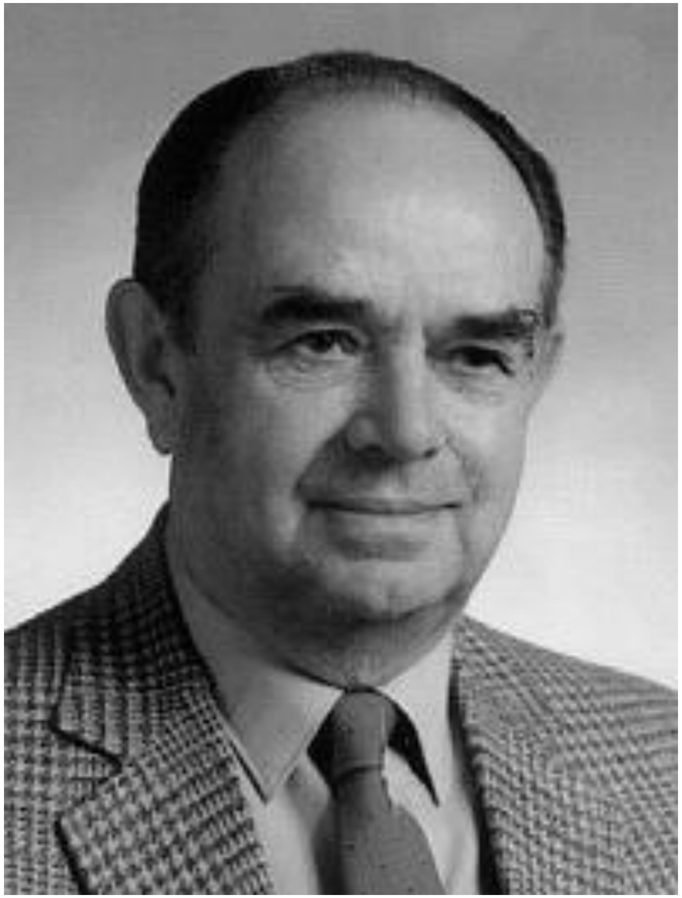
Reputed Scientists

Boris Vladimirovich Gnedenko, 1912-1995

Reputed Scientists

Leonard Kleinrock, 1934 -

Hungarian Contributions



Lajos Takács, 1924 -

Takács Formulas, M/G/1 Sytems

$$\mathbb{E}(W^k) = \frac{\lambda}{1-\rho}\sum_{i=1}^{k}\binom{k}{i}\frac{\mathbb{E}(S^{i+1})}{i+1}\mathbb{E}(W^{k-i})$$

$$\mathbb{E}(T^k) = \sum_{l=0}^{k}\binom{n}{l}\mathbb{E}(W^l)\cdot\mathbb{E}(S^{k-l})$$

$$L_\delta(t) = L_S(t+\lambda-\lambda L_\delta(t))$$

Hungarian Contributions

- Eötvös Loránd University
  (A. Benczúr, L. Lakatos, L. Szeidl)
- Budapest University of Technology and Economics
  (L. Györfi, M. Telek, S. Molnár)
- University of Debrecen
  (J. Tomkó, M. Arató, B. Almási, A. Kuki, J. Sztrik)

Java Applets and Information

- http://irh.inf.unideb.hu/user/jsztrik/education/09/index.html

http://irh.inf.unideb.hu/user/jsztrik/

Tool supported modeling

- University of Dortmund: *HIT, HiQPN, APNN*
  http://ls4-www.infromatik.uni-dortmund.de/tools.html/
- University of Illinois at Urbana-Champaign: *MÖBIUS*
  http://www.mobius.uiuc.edu/
- University of Erlangen: *PEPSY, MOSEL*
  http://www4.informatik.uni-erlangen.de/Projects/MOSEL/
- University of Oxford: *PRISM*
  http://www.prismmodelchecker.org/

Softwares and Information

http://web2.uwindsor.ca/math/hlynka/qsoft.html

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- GNEDENKO, B.V. – KOVALENKO I.N. *Introduction to Queueing Theory, Second Edition*, Birkhauser, 1989
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- KHINTCHINE, A.Y. *Mathematical Methods in the Theory of Queueing, Second Edition*, Hafner Publication Company, 1969
- KLEINROCK, L. *Queueing Systems, Vol. I-II*, John Wiley Sons, 1976
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- DATTATREYA, G. *Performance Analysis of Queueing and Computer Networks*, CRC Press, 2008
- JAIN, R. *The Art of Computer Systems Performance Analysis*, John Wiley Sons, 1991
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- TRIVEDI, K. *Probability and Statistics with Reliability, Queueing, and Computer Science Applications*, John Wiley Sons, 2002

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- CAI, L., SHEN, X., MARK, J.W. *Multimedia Services in Wireless Internet, Modeling and Analysis*, John Wiley Sons, 2009
- GEBALI, F. *Analysis of Computer and Communication Networks*, Springer, 2008
- KOUVATSOS, D. *Network Performance Engineering, A Handbook on Convergent Multi-Service Networks and Next Generation Internet*, Springer, 2011
- MISIS, J., MISIC, V.B. *Performance Modeling and Analysis of Bluetooth Networks: Polling, Scheduling and Traffic Control*, Auerbach Publications, 2006

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Heterogeneous finite-source retrial queueing system

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HETEROGENEOUS FINITE-SOURCE RETRIAL QUEUEING SYSTEMS

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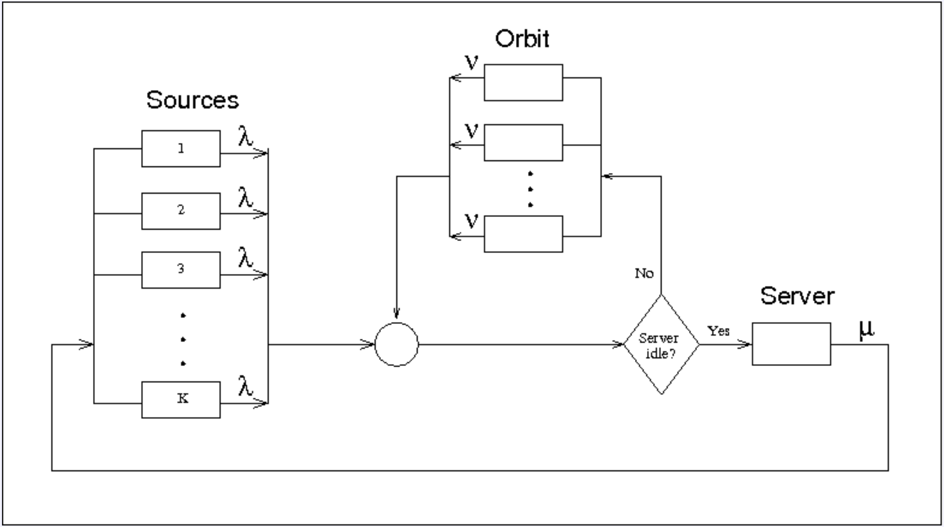
Outline

The queueing model



Applications

- magnetic disk memory systems
- local area networks with CSMA/CD protocols
- collision avoidance local area network modeling

Mathematical model

$$P(0;0) = \lim_{t\to\infty} P(C(t) = 0, N(t) = 0)$$

$$P(j;0) = \lim_{t\to\infty} P(\alpha_1 = j, N(t) = 0), \;\; j = 1, ..., K$$

$$P(0;i_1,...,i_k) = \lim_{t\to\infty} P(C(t) = 0, \beta_1 = i_1, ..., \beta_k = i_k), \;\; k = 1, ..., K-1$$

$$P(j;i_1,...,i_k) = \lim_{t\to\infty} P(\alpha_1 = j, \beta_1 = i_1, ..., \beta_k = i_k), \;\; k = 1, ..., K-1.$$

Performance measures

Once we have obtained these limiting probabilities the **main system performance measures** can be derived in the following way.

1. The server utilization with respect to source j

$$U_j = \text{P (the server is busy with source } j \text{)}$$

that is, we have to summarize all the probabilities where the first component is j. Formaly

$$U_j = \sum_{k=0}^{K-1} \sum_{i_1,\dots,i_k \neq j} P(j; i_1, \dots, i_k).$$

Hence the **server utilization**

$$U = E[C(t) = 1] = \sum_{j=1}^{K} U_j.$$

Let us denote by $P_W^{(i)}$ the steady state probability that request i is waiting (staying in the orbit). It is easy to see that

$$P_W^{(i)} = \sum_{j=0, j\neq i}^{K} \sum_{k=1}^{K-1} \sum_{i \epsilon (i_1, \ldots, i_k)} P(j; i_1, \ldots, i_k).$$

Similarly, it can easily be seen, that the steady state probability $P^{(i)}$ that request i is in the service facility (it is under service or waiting in the orbit) is given by

$$P^{(i)} = P_W^{(i)} + U_i.$$

2. Mean response time of source i

Let us denote by $E[T_i]$ the mean response time of customer i, and by γ_i the **throughput** of request i, that is, the mean number of times that request i is served per unit time. These are related by

$$\gamma_i = \frac{1}{E[T_i] + 1/\lambda_i} = \lambda_i(1 - P^{(i)}) = \mu_i U_i, \qquad i = 1, ..., K. \quad (1)$$

For $P^{(i)}$ we have

$$P^{(i)} = \frac{E[T_i]}{E[T_i] + 1/\lambda_i} = \gamma_i E[T_i] = 1 - \frac{\gamma_i}{\lambda_i} \qquad i = 1, ..., K. \quad (2)$$

which represents **Little's theorem** for request i in the service facility. It is easy to see that as a consequence of (1) we have

$$P^{(i)} = 1 - \frac{\mu_i}{\lambda_i} U_i,$$

and

$$P_W^{(i)} = P^{(i)} - U_i = 1 - \frac{\mu_i + \lambda_i}{\lambda_i} U_i.$$

Alternatively, by the help of (2) we can express the mean response time $E[T_i]$ for request i in terms of U_i as

$$E[T_i] = \frac{P^{(i)}}{\lambda_i(1 - P^{(i)})} = \frac{1 - \frac{\mu_i}{\lambda_i} U_i}{\mu_i U_i} = \frac{\lambda_i - \mu_i U_i}{\lambda_i \mu_i U_i}. \qquad (3)$$

3. Mean waiting time of source i

The mean waiting time of request i is given by

$$E[W_i] = E[T_i] - \frac{1}{\mu_i} = \frac{1}{\gamma_i} - \frac{1}{\lambda_i} - \frac{1}{\mu_i} = \frac{\lambda_i - (\mu_i + \lambda_i) U_i}{\lambda_i \mu_i U_i}. \qquad (4)$$

At the same time we have another **Little's theorem** for request i waiting for service. Namely

$$P_W^{(i)} = \frac{E[W_i]}{E[T_i] + 1/\lambda_i} = \gamma_i E[W_i] \qquad i = 1, ..., K.$$

4. Mean number of calls staying in the orbit or in service

$$M = E[C(t) + N(t)] = \sum_{i=1}^{K} P^{(i)} = \sum_{i=1}^{K} (1 - \frac{\mu_i}{\lambda_i} U_i) = K - \sum_{i=1}^{K} \frac{\mu_i}{\lambda_i} U_i.$$

5. Mean number of sources of repeated calls

$$N = E[N(t)] = \sum_{i=1}^{K} P_W^{(i)} = \sum_{i=1}^{K} (1 - \frac{\mu_i + \lambda_i}{\lambda_i} U_i) = K - \sum_{i=1}^{K} \frac{\mu_i + \lambda_i}{\lambda_i} U_i.$$

6. Mean rate of generation of primary calls

$$\overline{\lambda} = \sum_{i=1}^{K} \gamma_i = \sum_{i=1}^{K} \lambda_i (1 - P^{(i)}) = \sum_{i=1}^{K} \mu_i U_i.$$

7. Blocking probability of primary call i

$$B_i = \frac{\lambda_i \sum_{j=1, j \neq i}^{K} \sum_{k=0}^{K-1} \sum_{i \neq i_1, \ldots, i_k} P(j; i_1, \ldots, i_k)}{\overline{\lambda}}.$$

Hence **blocking probability of primary calls**

$$B = \sum_{i=1}^{K} B_i.$$

In particular, in the case of **homogeneous calls**

$$U_i = E[C(t)]/K, \quad i = 1, ..., K, \quad N = K - \frac{(\lambda + \mu)U}{\lambda},$$

$$\overline{\lambda} = \lambda E[K - C(t) - N(t)] = \mu U,$$

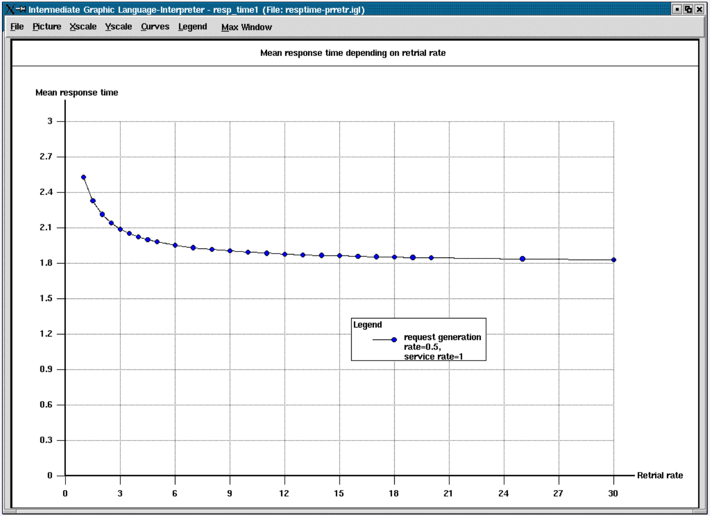
$$E[W] = \frac{N}{\overline{\lambda}} = K - \frac{1}{\mu U} - \frac{1}{\lambda} - \frac{1}{\mu},$$

$$B = \frac{\lambda E[K - C(t) - N(t); C(t) = 1]}{\lambda E[K - C(t) - N(t)]}.$$

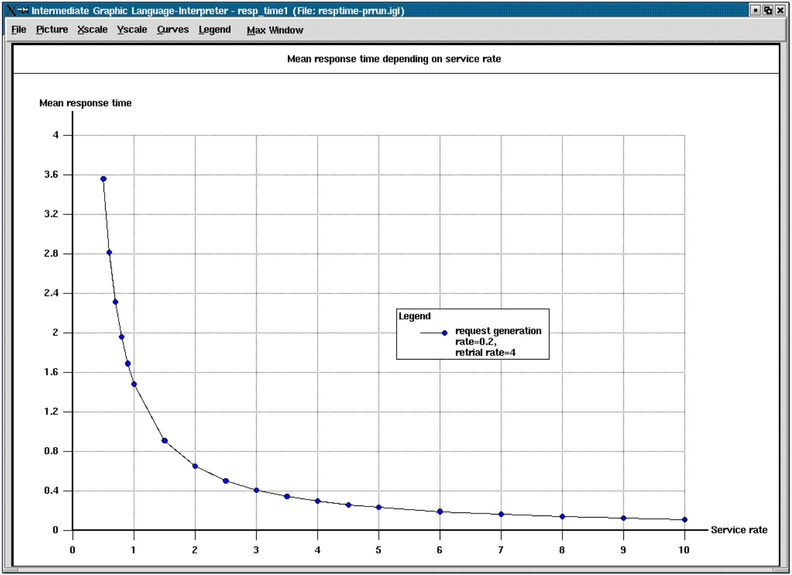
Evaluation Tool MOSEL

MOSEL (Modeling, Specification and Evaluation Language) developed at the University of Erlangen, Germany, is used to formulate and solved the problem.

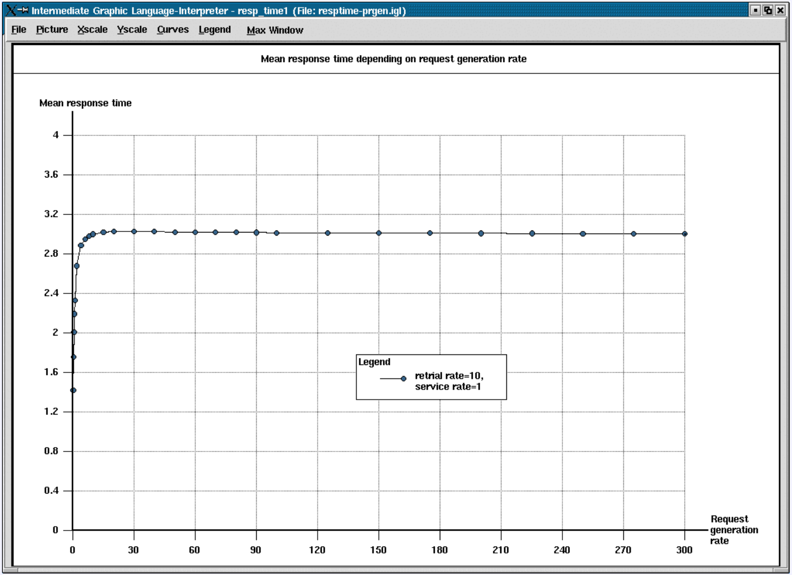
Case studies



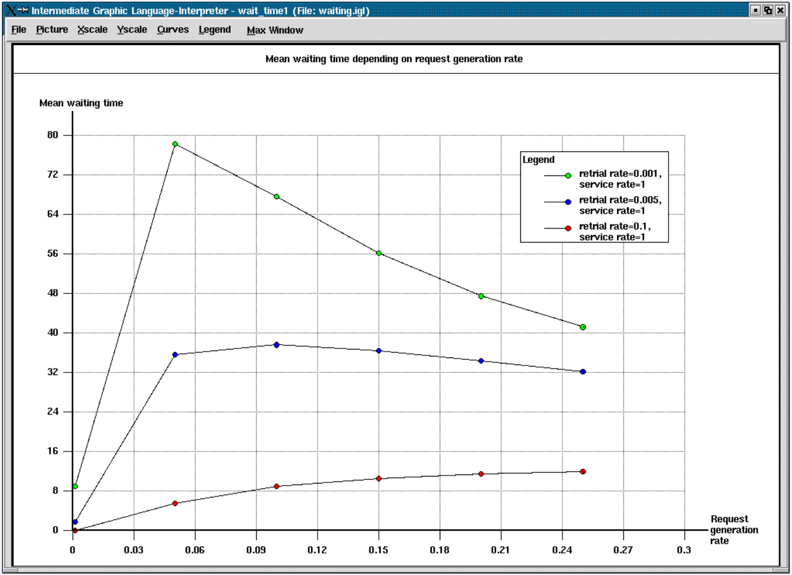
$E[T]$ versus retrial rate



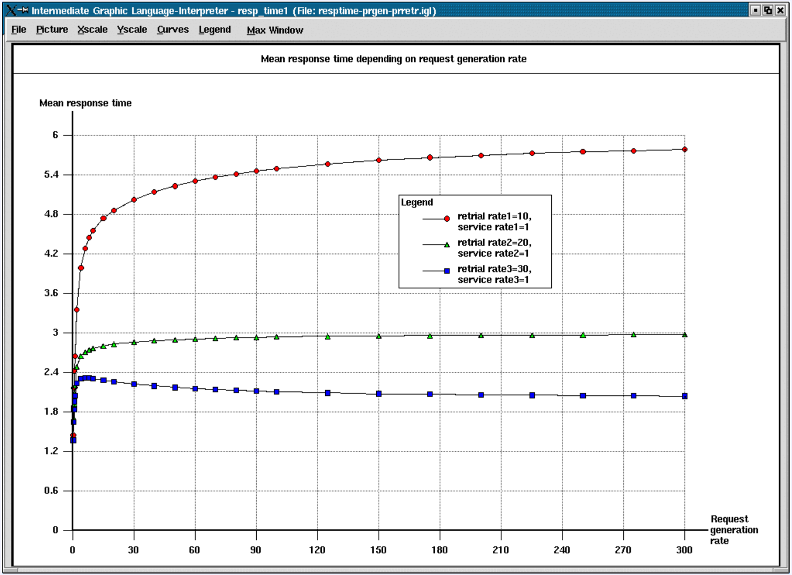
$E[T]$ versus service rate



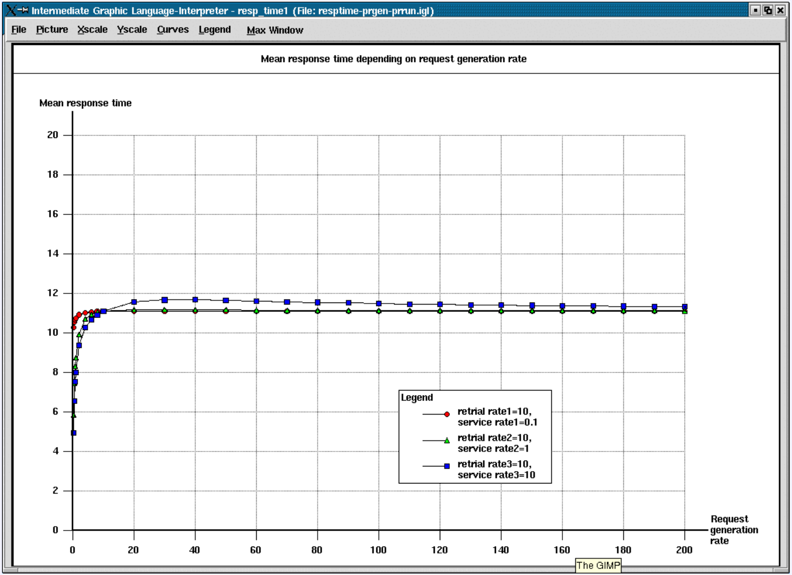
$E[T]$ versus primary request generation rate



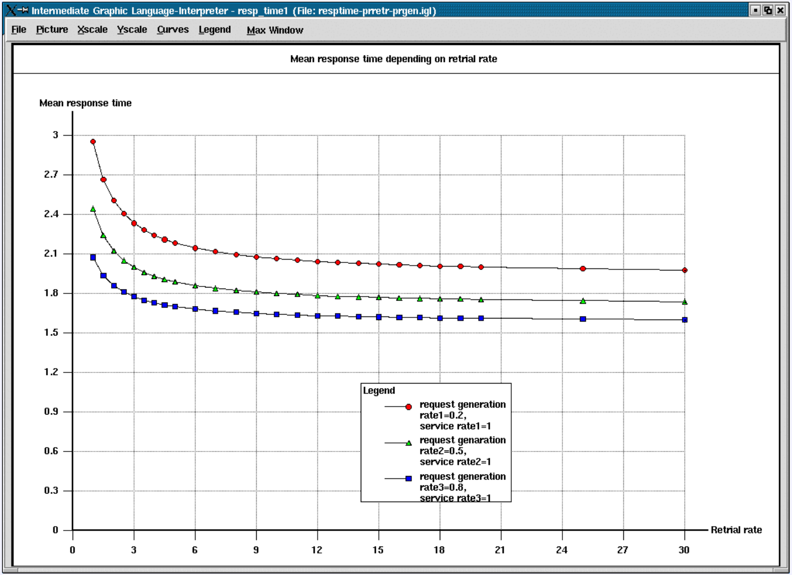
$E[T]$ versus primary request generation rate



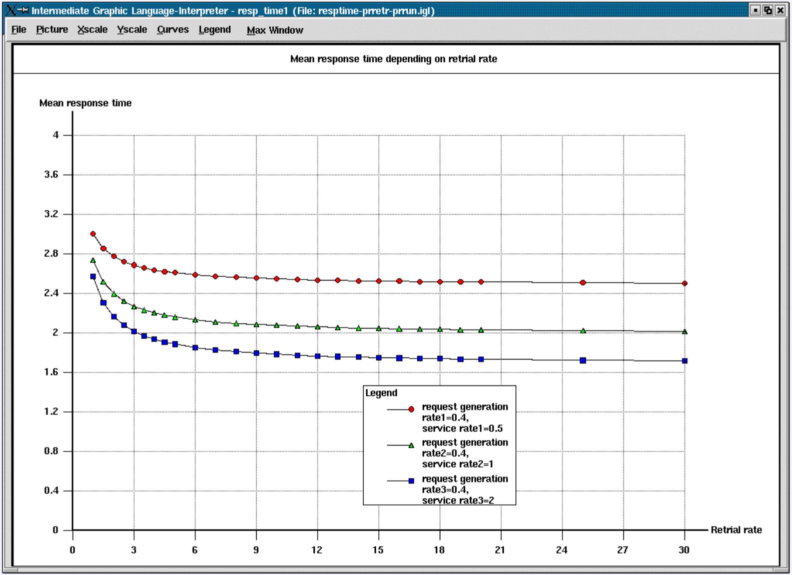
$E[T]$ versus primary request generation rate with homogeneous service and heterogeneous retrial



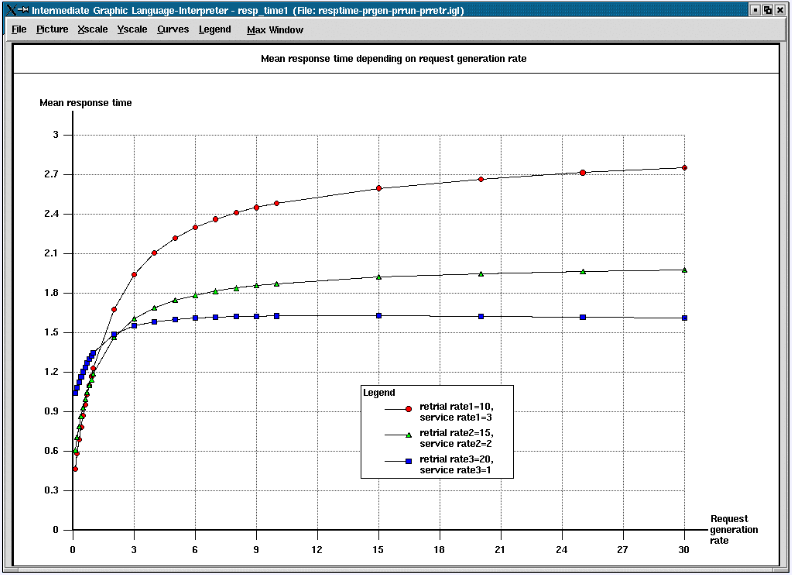
$E[T]$ versus primary request generation rate with homogeneous retrial and heterogeneous service



$E[T]$ versus retrial rate with homogeneous service and heterogeneous primary request generation



$E[T]$ versus retrial rate with homogeneous primary request generation and heterogeneous service



$E[T]$ versus primary request generation rate with heterogeneous service and heterogeneous retrial

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Thank You
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Attention